Section 2.3

1. In Leibniz notation the derivative is dD/dt and the units are feet per minute.

3. In Leibniz notation the derivative is dN/dD and the units are gallons per mile.

5.

(a) The statement $f(5) = 18$ means that when 5 milliliters of catalyst are present, the reaction will take 18 minutes. Thus, the units for 5 are ml while the units for 18 are minutes.

(b) As in part (a), 5 is measured in ml. Since f' tells how fast T changes per unit a, we have f' measured in minutes/ml. If the amount of catalyst increases by 1 ml (from 5 to 6 ml), the reaction time decreases by about 3 minutes.

7.

(a) The 12 represents the weight of the chemical; therefore, its units are pounds. The 5 represents the cost of the chemical; therefore, its units are dollars. The statement $f(12) = 5$ means that when the weight of the chemical is 12 pounds, the cost is 5 dollars.

(b) We expect the derivative to be positive since we expect the cost of the chemical to increase when the weight bought increases.

(c) Again, 12 is the weight of the chemical in pounds. The units of the 0.4 are dollars/pound since it is the rate of change of the cost as a function of the weight of the chemical bought. The statement $f'(12) = 0.4$ means that the cost is increasing at a rate of 0.4 dollars per pound when the weight is 12 pounds, or that an additional pound will cost about an extra 40 cents.

11.

(a) The yam is cooling off so T is decreasing and $f'(t)$ is negative.

(b) Since $f(t)$ is measured in degrees Fahrenheit and t is measured in minutes, df/dt must be measured in units of $^\circ$F/min.

17.

(a) The statement $f(200) = 1300$ means that it costs \$1300 to produce 200 gallons of the chemical.

(b) The statement $f'(200) = 6$ means that when the number of gallons produced is 200, costs are increasing at a rate of \$6 per gallon. In other words, it costs about \$6 to produce the next (the 201^{st}) gallon of the chemical.

19.

(a) Positive, since weight increases as the child gets older.

(b) $f(8) = 45$ tells us that when the child is 8 years old, the child weighs 45 pounds.

(c) The units of $f'(a)$ are lbs/year. $f'(a)$ tells the rate of growth in lbs/years at age a.

(d) $f'(8) = 4$ tells us that the 8-year-old child is growing at a rate of 4 lbs/year.

(e) As a increases, $f'(a)$ will decrease since the rate of growth slows down as the child grows up.

29.

(a) The statement $f(140) = 120$ means that a patient weighing 140 pounds should receive a dose of 120 mg of the painkiller. The statement $f'(140) = 3$ tells us that if the weight of a patient increases by one pound (from 140 pounds), the dose should be increased by about 3 mg.

(b) Since the dose for a weight of 140 lbs is 120 mg and at this weight the dose goes up by about 3 mg for one pound, a 145 lb patient should get about an additional $3(5) = 15$ mg. Thus, for a 145 lb patient, the correct dose is approximately

$$f(145) \approx 120 + 3(5) = 135 \text{ mg}.$$

48 . Estimating the relative rate of change using $\Delta t = 0.01$, we have

$$\frac{1}{f}\frac{\Delta f}{\Delta t} \approx \frac{1}{f(4)}\frac{f(4.01) - f(4)}{0.01} = \frac{1}{4^2}\frac{4.01^2 - 4^2}{0.01} = 0.50.$$

51. **(a)** At 2 1/2 months, the baby weighs 5.67 kilograms.

(b) At 2 1/2 months, the baby's weight is increasing at a relative rate of 13% per month.

52. (a) By June 2011, 15 billion of downloads with monthly average of 0.93 billion.

(b) The relative rate of change of P at t=36 is

$$\frac{g'(36)}{g(36)} = \frac{0.93}{15} = 6.2\%$$

In June 2011, downloads was increasing at a continuous rate of 6.2%