44 Surface Area

The **surface area** of a space figure is the total area of all the faces of the figure. In this section, we discuss the surface areas of some of the space figures introduced in Section 41.

Right Prisms

Let's find the surface area of the right prism given in Figure 44.1.

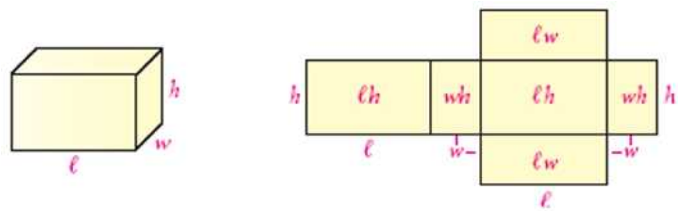


Figure 44.1

As you can see, we can dissamble the prism into six rectangles. The total area of these rectangles which is the surface area of the box is

$$SA = 2lw + 2h(w + l).$$

If we let A denote the area of the base and P the perimeter of the base then $A = lw$ and $P = 2(l + w)$. Thus,

$$SA = 2A + Ph.$$

This formula is valid for any right prism with height h, area of base A, and perimeter of base P.

Right Cylinders

To find the surface area of a right cylinder with height h and radius of base r we dissamble the cylinder into a rectangle and two circles as shown in Figure 44.2

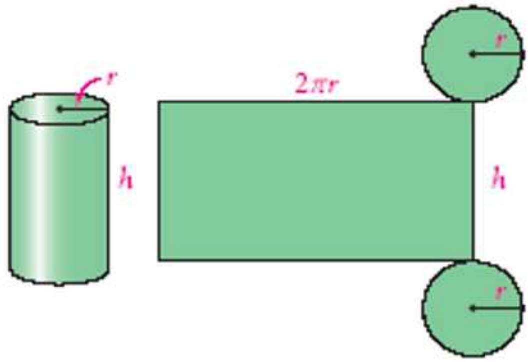


Figure 44.2

Thus, the surface area is the sum of the area of the rectangle together with the areas of the two circles. That is,

$$SA = 2\pi r^2 + 2\pi r \cdot h = 2\pi r(r + h).$$

Example 44.1
A thin can has a height of 15 cm and a base radius of 7 cm. It is filled with water. Find the total surface area in contact with the water. (Take $\pi = 22/7$)

Solution.

$$SA = 2\pi(7)(7 + 15) = 2 \cdot \frac{22}{7} \cdot 7 \cdot 22 = 968\ cm^2 \blacksquare$$

Example 44.2
The lateral surface area of a solid cylinder is 880 cm^2 and its height is 10 cm. Find the circumference and area of the base of the cylinder. Take $\pi = \frac{22}{7}$.

Solution.
Since the lateral surface of a cylinder is height $\times$ perimeter of the base then $P = \frac{880}{10} = 88\ cm$. Now, the radius of the circle is $r = \frac{88}{2\pi} = 44 \cdot \frac{7}{22} = 14\ cm$. Finally, the area of the base is $A = \pi 14^2 = \frac{22}{7} \cdot 196 = 616\ cm^2 \blacksquare$

Pyramids
The surface area of a pyramid is obtained by summing the areas of the faces.

We illustrate this for the square pyramid shown in Figure 44.3

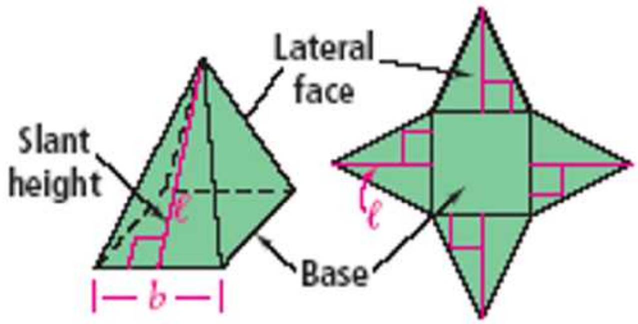


Figure 44.3

The area of the base is b^2 whereas the total area of the lateral faces is $4 \times \frac{bl}{2} = 2bl$. Thus,

$$SA = b^2 + 2bl.$$

If we let A denote the area of the base and P the perimeter of the base then

$$SA = A + \frac{1}{2}Pl.$$

This formula is valid for any right pyramid. We call l the **Slant height**.

Example 44.3
A right pyramid with slant height of 12 cm stands on a square base of sides 10 cm. Calculate the total surface area.

Solution.

$$SA = 10^2 + 2(10)(12) = 100 + 240 = 340\ cm^2\ \blacksquare$$

Right Cones
First, we define the **slant height** of a right circular cone to be the length of a straight line drawn from any point on the perimeter of the base to the apex. See Figure 44.4. If the radius of the base is r and the altitude of the cone is h, then by applying the Pythagorean formula we find that the length of the slant height is given by

$$l = \sqrt{r^2 + h^2}$$

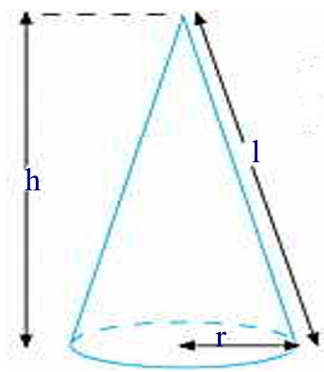


Figure 44.4

To find the surface area of the cone, we cut it along a slant height and open it out flat to obtain a circular sector shown in Figure 44.5

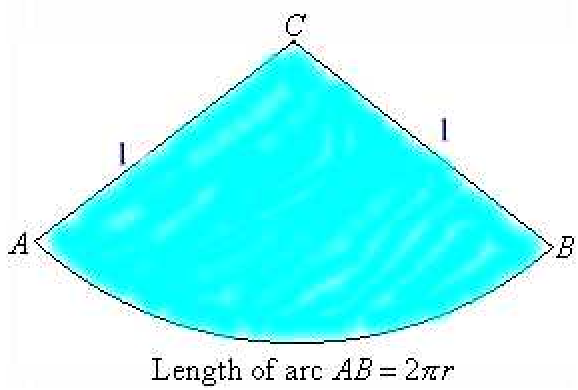


Figure 44.5

Thus, the lateral surface area is the area of the circular sector. But

$$\begin{aligned}
\frac{\text{Area of sector ABC}}{\text{Area of circle with center C}} &= \frac{\text{Arc length of AB}}{\text{Circumference of circle with center at C}} \\
\frac{\text{Area of sector ABC}}{\pi l^2} &= \frac{2\pi r}{2\pi l} \\
\text{Area of sector ABC} &= \pi r l
\end{aligned}$$

But

$$\text{Total surface area} = \text{Area of base} + \text{lateral surface area.}$$

Hence

$$\begin{aligned} SA &= \pi r^2 + \pi r l \\ &= \pi r(r+l) \\ &= \pi r(r + \sqrt{r^2+h^2}) \end{aligned}$$

Spheres
Archimedes discovered that a cylinder that circumscribes a sphere, as shown in Figure 44.6, has a lateral surface area equal to the surface area, SA, of the sphere.

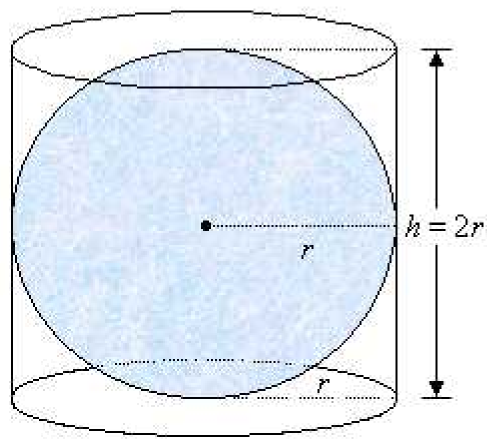


Figure 44.6

Thus,

$$\begin{aligned} SA &= 2\pi r h \\ &= 2\pi r \times 2r \\ &= 4\pi r^2 \end{aligned}$$

Example 44.4
A solid sphere has a radius of 3 m. Calculate its surface area. Round your answer to a whole number. (Take $\pi = 22/7$)

Solution.

$$\begin{aligned} SA &= 4\pi r^2 \\ &= 4 \times \frac{22}{7} \times 3^2 \\ &\approx 113\ m^2 \ \blacksquare \end{aligned}$$

Example 44.5
Find the radius of a sphere with a surface area of $64\pi\ m^2$.

Solution.

$$\begin{aligned} SA &= 4\pi r^2 \\ 64\pi &= 4\pi r^2 \\ 16 &= r^2 \\ r &= 4\ cm \blacksquare \end{aligned}$$

Practice Problems

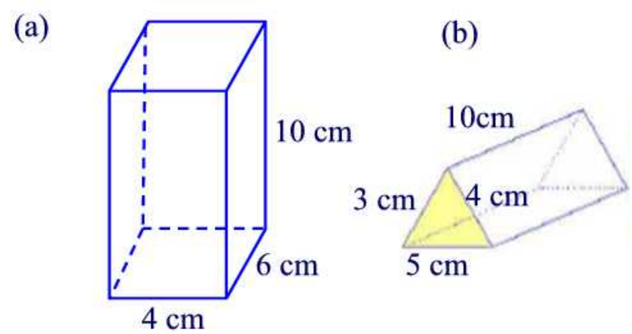
Problem 44.1
A small can of frozen orange juice is about 9.5 cm tall and has a diameter of about 5.5 cm. The circular ends are metal and the rest of the can is cardboard. How much metal and how much cardboard are needed to make a juice can?

Problem 44.2
A pyramid has a square base 10 cm on a side. The edges that meet the apex have length 13 cm. Find the slant height of the pyramid, and then calculate the total surface area of the pyramid.

Problem 44.3
An ice cream cone has a diameter of 2.5 in and a slant height of 6 in. What is the lateral surface area of the cone?

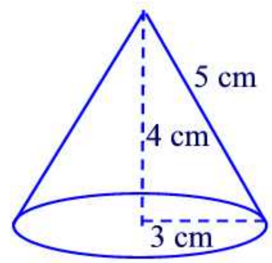
Problem 44.4
The diameter of Jupiter is about 11 times larger than the diameter of the planet Earth. How many times greater is the surface area of Jupiter?

Problem 44.5
Find the surface area of each of the right prisms below.



Problem 44.6
The Great Pyramid of Cheops is a right square pyramid with a height of 148 m and a square base with perimeter of 930 m. The slant height is 188 m. The basic shape of the Transamerica Building in San Fransisco is a right square pyramid that has a height of 260 m and a square base with a perimeter of 140 m. The altitude of slant height is 261 m. How do the lateral surface areas of the two structures compare?

Problem 44.7
Find the surface area of the following cone.



Problem 44.8
The Earth has a spherical shape of radius 6370 km. What is its surface area?

Problem 44.9
Suppose one right circular cylinder has radius 2 m and height 6 m and another has radius 6 m and height 2 m.

(a) Which cylinder has the greater lateral surface area?
(b) Which cylinder has the greater total surface area?

Problem 44.10
The base of a right pyramid is a regular hexagon with sides of length 12 m. The height of the pyramid is 9 m. Find the total surface area of the pyramid.

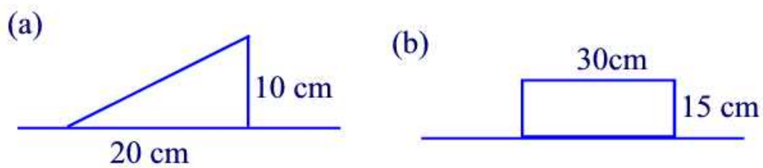
Problem 44.11
A square piece of paper 10 cm on a side is rolled to form the lateral surface area of a right circular cylinder and then a top and bottom are added. What is the surface area of the cylinder?

Problem 44.12
The top of a rectangular box has an area of 88 cm^2. The sides have areas 32 cm^2 and 44 cm^2. What are the dimensions of the box?

Problem 44.13
What happens to the surface area of a sphere if the radius is tripled?

Problem 44.14
Find the surface area of a square pyramid if the area of the base is 100 cm^2 and the height is 20 cm.

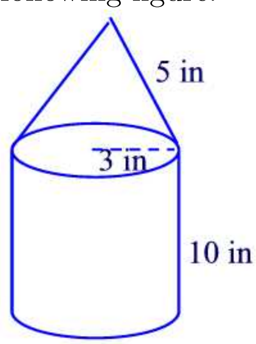
Problem 44.15
Each region in the following figure revolves about the horizontal axis. For each case, sketch the three dimensional figure obtained and find its surface area.



Problem 44.16
The total surface area of a cube is 10,648 cm^2. Find the length of a diagonal that is not a diagonal of a face.

Problem 44.17
If the length, width, and height of a rectangular prism is tripled, how does the surface area change?

Problem 44.18
Find the surface area of the following figure.



Problem 44.19
A room measures 4 meters by 7 meters and the ceiling is 3 meters high. A liter of paint covers 40 square meters. How many liters of paint will it take to paint all but the floor of the room?

Problem 44.20
Given a sphere with diameter 10, find the surface area of the smallest cylinder containing the sphere.