42 Systems of Measurements

So far only two goemetric figures have been measured: line segments, measured by the distance between their endpoints; and angles, measured by the degrees of rotation needed to turn one side to the other. In this and coming sections we will introduce more general notions of measurement of geometric figures. All curves, not just segments will be given a length. Plane regions will be measured by area and perimeter. Space figures will be measured by surface area and volume.
We begin by discussing the general process of measurement. The two principal systems of measurements are then described: the English System and the Metric or International System.

The Measurement Process
The measurement process consists of the following steps:
(1) Select an object and an attribute of the object to measure, such as its length, area, volume, temperature, or weight.
(2) Select an appropriate unit which to measure the attribute.
(3) Determine the number of units needed to measure the attribute.

Measurements can be done using either nonstandard units or standard units. For example, the first nonstandard units of length were based on people's hands, feet, and arms. Since different people have different-sized hands, feet, and arms then these measurements are adequate for convenient but lack accuracy. This is why we need standard measuring units such as of those of the metric system. By using standard units, people around the globe communicate easily with one another about measurements. We consider two standard systems of measurements:

The US Customary System(English System)
The following are the common conversion equivalencies in the English System.
Length

$$1 \text{ in (inch)} = \tfrac{1}{12} \text{ ft (foot)}$$
$$1 \text{ yd (yard)} = 3 \text{ ft}$$
$$1 \text{ rd (rod)} = 16.5 \text{ ft}$$
$$1 \text{ furlong} = 660 \text{ ft}$$

1 mi (mile) = 5280 ft

Area

1 square inch = $\frac{1}{144}$ square feet
1 square yard = 9 square feet
1 acre = 43,560 square feet
1 square mile = 27,878,400 square feet

Volume

1 cubic in = $\frac{1}{1728}$ cubic feet
1 cubic yard = 27 cubic feet

Capacity

1 tablespoon = 3 teaspoons
1 ounce = 2 tablespoons
1 cup = 8 ounces
1 pint = 2 cups
1 quart = 2 pints
1 gallon = 4 quarts
1 barrel = 31.5 gallons

Weight

1 pound = 16 ounces
1 ton = 2000 pounds

Time

1 minute = 60 seconds
1 hour = 60 minutes
1 day = 24 hours
1 week = 7 days
1 year $\approx$ 365.25 days

Temperature(Fahrenheit)

$32^\circ F$ = freezing point of water
$212^\circ F$ = boiling point of water

Dimensional Analysis
To convert from one unit to another, the process knwon as **dimensional analysis** can be used. This process works with unit ratios(ratios equal to 1). For example, $\frac{1\ yd}{3\ ft}$ and $\frac{5280\ ft}{1\ mi}$ are units ratios. Therefore to convert 5.25 mi to yards, we have the following

$$5.25 \text{ mi} = 5.25\ mi \times \frac{5280\ ft}{1\ mi} \times \frac{1\ yd}{3\ ft} = 9240 \text{ yd}$$

Example 42.1
If a cheetah is clocked at 60 miles per hour, what is its speed in feet per second?

Solution.

$$60\frac{mi}{hr} = 60\frac{mi}{hr} \times \frac{5280\ ft}{1\ mi} \times \frac{1\ hr}{60\ min} \times \frac{1\ min}{60\ sec} = 88\frac{ft}{sec}\ \blacksquare$$

Example 42.2
Convert each of the following:
(a) 219 ft = ____ yd
(b) 8432 yd = ____ mi
(c) 0.2 mi = ____ ft
(d) 64 in = ____ yd

Solution.
(a) 219 ft = 219 ft $\times\frac{1\ yd}{3\ ft} = 73$ yd.
(b) 8432 yd = 8432 yd $\times\frac{3\ ft}{1\ yd} \times \frac{1\ mi}{5280\ ft} \approx 4.79\ mi.$
(c) 0.2 mi = 0.2 mi $\times\frac{5280\ ft}{1\ mi} = 1056$ ft
(d) 64 in = 64 in $\times\frac{1\ ft}{12\ in} \times \frac{1\ yd}{3\ ft} \approx 1.78$ yd $\blacksquare$

International System
Six common prefixes used in the metric system are listed below.

Prefixes	Multiple or fraction
kilo	1000
hecto	100
deca	10
deci	0.1
centi	0.01
milli	0.001

The following are the common conversion equivalencies in the Metric System.

Length

Unit	Abbreviation	Number of Meters
kilometer	km	1,000
hectometer	hm	100
dekameter	dam	10
meter	m	1
decimeter	dm	0.1
centimeter	cm	0.01
millimeter	mm	0.001

Area

Unit	Abbreviation	Number of square meters
square millimeter	mm^2	0.000001
square centimeter	cm^2	0.0001
square decimeter	dm^2	0.01
Are	a	100
Hectare	ha	10000
square kilometer	km^2	1000000

Volume

Unit	Abbreviation	Number of cubic meters
cubic meter	m^3	1
cubic decimeter	dm^3	0.001
cubic centimeter	cm^3	0.000001

Capacity

Unit	Abbreviation	Number of liters
kiloliter	kL	1,000
hectoliter	hL	100
dekaliter	daL	10
liter	L	1
deciliter	dL	0.10
centiliter	cL	0.01
milliliter(cm^3)	mL	0.001

Mass

Unit	Abbreviation	Number of grams
metric ton	t	1,000,000
kilogram	kg	1,000
hectogram	hg	100
dekagram	dag	10
gram	g	1
decigram	dg	0.10
centigram	cg	0.01
milligram	mg	0.001

Temperature(Celsius)

$0°C$ = freezing point of water
$100°C$ = boiling point of water

Example 42.3
Convert the following measurements to the unit shown.

(a) 1495 mm = ____ m
(b) 29.4 cm = ____ mm
(c) 38741 m = ____ km

Solution.
(a) 1495 mm = 1495 $mm \times \frac{1\ m}{1000\ mm}$ = 1.495 m
(b) 29.4 cm = 29.4 $cm \times \frac{1\ mm}{0.1\ cm}$ = 294 mm
(c) 38741 m = 38741 $m \times \frac{1\ km}{1000\ m}$ = 38.741km ■

Conversion Between Systems
Length: 1 in = 2.54 cm
Area: 1 $in^2 = 2.54^2\ cm^2$
Volume: 1 $in^3 = 2.54^3\ cm^3$
Capacity: 1 cm^3 = 1 mL, 1 gal ≈ 3.79 L
Weight: 1 oz = 29 g
Temperature: $F = \frac{9}{5}C + 32$

Practice Problems

Problem 42.1
A small bottle of Perrier sparking water contains 33 cl. What is the volume in ml?

Problem 42.2
Fill in the blanks.
(a) 58728 g = ____ kg
(b) 632 mg = ____ g
(c) 0.23 kg = ____ g

Problem 42.3
Convert each of the following.
(a) 100 in = ____ yd
(b) 400 yd = ____ in
(c) 300 ft = ____ yd
(d) 372 in = ____ ft

Problem 42.4
Complete the following table.

Item	m	cm	mm
Length of a piece of paper		35	
Height of a woman	1.63		
Width of a filmstrip			35
Length of a cigarette			100

Problem 42.5
List the following in decreasing order: 8 cm, 5218 mm, 245 cm, 91 mm, 6 m, 700 mm.

Problem 42.6
Complete each of the following.
(a) 10 mm = ____ cm
(b) 3 km = ____ m
(c) 35 m = ____ cm
(d) 647 mm = ____ cm

Problem 42.7
Complete the following conversions.
(a) 3 feet = ____ inches
(b) 2 miles = ____ feet
(c) 5 feet = ____ yards

Problem 42.8
Complete the following conversions.
(a) 7 yards = ____ feet
(b) 9 inches = ____ feet
(c) 500 yards = ____ miles

Problem 42.9
Complete the following conversions.
(a) 9.4 L = ____ mL
(b) 37 mg = ____ g
(c) 346 mL = ____ L

Problem 42.10
A nurse wants to give a patient 0.3 mg of a certain drug. The drug comes in a solution containing 0.5 mg per 2 mL. How many milliters should be used?

Problem 42.11
A nurse wants to give a patient 3 gm of sulfisoxable. It comes in 500 mg tablets. How many tablets should be used?

Problem 42.12
Complete the following conversions.
(a) 3 gallons = ____ quarts
(b) 5 cups = ____ pints
(c) 7 pints = ____ quarts
(d) 12 cups = ____ gallons

Problem 42.13
True or false? Expain.
(a) 1 mm is longer than 1 in.
(b) 1 m is longer than 1 km.
(c) 1 g is heavier than 1 lb.
(d) 1 gallon is more than 1 L.

Problem 42.14
Derive a conversion formula for degrees Celsius to degrees Fahrenheit.

Problem 42.15
A temperature of $-10°C$ is about
(a) $-20°F$ (b) $10°F$ (c) $40°F$ (d) $70°F$

Problem 42.16
Convert the following to the nearest degree
(a) Moderate oven ($350°F$) to degrees Celsius.
(b) $20°C$ to degrees Fahrenheit.
(c) $-5°C$ to degrees Fahrenheit.

Problem 42.17
Complete the following conversions.
(a) 1 cm^2 = ____ mm^2
(b) 610 dam^2 = ____ hm^2
(c) 564 m^2 = ____ km^2
(d) 0.382 km^2 = ____ m^2

Problem 42.18
Suppose that a bullet train is traveling 200 mph. How many feet per seconds is it traveling?

Problem 42.19
A pole vaulter vaulted 19ft4$\frac{1}{2}$in. Find the height in meters.

Problem 42.20
The area of a rectangular lot is 25375 ft^2. What is the area of the lot in acres? Use the fact that 640 acres = 1 square mile.

Problem 42.21
A vase holds 4286 grams of water. What is the capacity in liters? Recall that the density of water is 1 g/cm^3.

Problem 42.22
By using dimensional analysis, make the following conversions.
(a) 3.6 lb to oz
(b) 55 mi/hr to ft/min
(c) 35 mi/hr to in/sec
(d) \$575 per day to dollars per minute.

Problem 42.23
The density of a substance is the ratio of its mass to its volume. A chunk of oak firewood weighs 2.85 kg and has a volume of 4100 cm^3. Determine the density of oak in g/cm^3, rounding to the nearest thousandth.

Problem 42.24
The speed of sound is 1100 ft/sec at sea level. Express the speed of sound in mi/hr.

Problem 42.25
What temperature is numerically the same in degrees Celsius and degrees Fahrenheit?