40 Regular Polygons

Convex and Concave Shapes

A plane figure is said to be **convex** if every line segment drawn between any two points inside the figure lies entirely inside the figure. A figure that is not convex is called a **concave** figure. Figure 40.1 shows a set of convex figures.

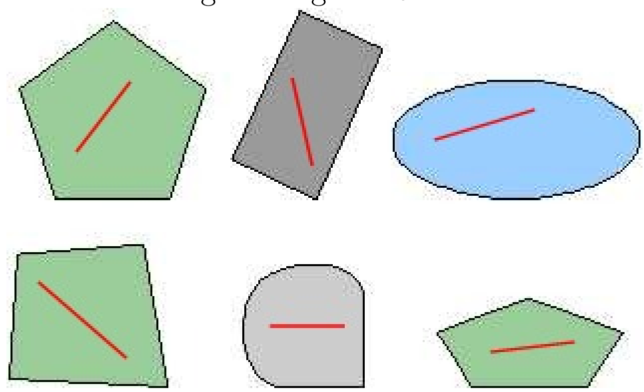


Figure 40.1

On the other hand, Figure 40.2 shows concave figures. To show that a figure is concave, it is enough to find two points within the figure whose corresponding line segment is not completely inside the figure.

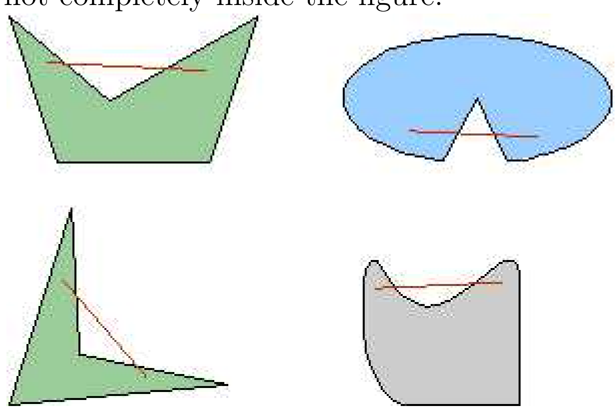


Figure 40.2

Regular Polygons

By a **closed** curve we mean a curve that starts from one point and ends in that same point. By a **simple** curve we mean a curve that does not cross itself. By a **simple closed curve** we mean a a curve in the plane that starts

and ends in the same location without crossing itself.
Several examples of curves are shown in Figure 40.3.

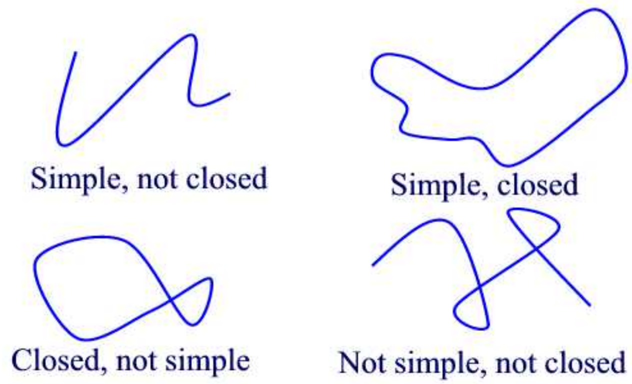


Figure 40.3

A **polygon** is a simple closed curve made up of line segments. A polygon whose line segments are congruent and whose interior angles are all congruent is called a **regular polygon**. If a regular polygon consists of n sides then we will refer to it as **regular n-gon**.
Figure 40.4 shows several tpyes of regular n-gons.

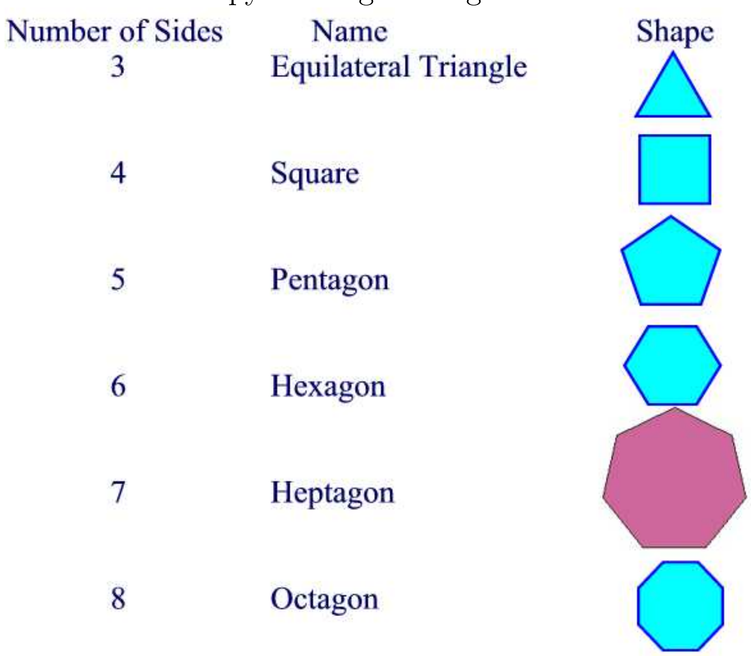


Figure 40.4

Angles of interest in a regular n-gon are the following: A **vertex angle** (also called an **interior** angle) is formed by two consecutive sides. A **central angle** is formed by the segments connecting two consecutive vertices to the center of the regular n-gon. An **exterior angle** is formed by one side together with the extension of an adjacent side as shown in Figure 40.5

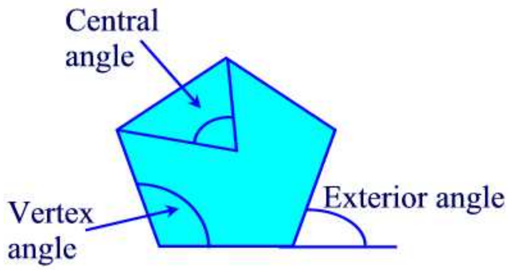
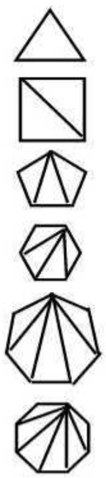


Figure 40.5

Angles Measures in Regular Polygons
Let's first find the measure of a central angle in a regular n-gon. Connecting the center of the n-gon to the n vertices we create n congruent central angles. Since the sum of the measures of the n central angles is 360° then the measure of each central angle is $\frac{360^\circ}{n}$.
Next, we will find the measure of each interior angle of a regular n-gon. We will use the method of recognizing patterns for that purpose. Since the angles are congruent then the measure of each is the sum of the angles divided by n. Hence, we need to find the sum of the interior angles. This can be achieved by dividing the n-gon into triangles and using the fact that the sum of the three interior angles in a triangle is 180°. The table below suggests a way for computing the measure of a vertex angle in a regular n-gon for n=3,4,5,6,7,8.



n	Sum of all interior angles(in degrees)	Measure of a vertex angle
3	$1.180 = 180^0$	$180/3 = 60^0$
4	$2.180 = 360^0$	$360/4 = 90^0$
5	$3.180 = 540^0$	$540/5 = 108^0$
6	$4.180 = 720^0$	$720/6 = 120^0$
7	$5.180 = 900^0$	$900/7 = 128\frac{4}{7}^0$
8	$6.180 = 1080^0$	$1080/8 = 135^0$

So, in general, the measure of an interior angle of a regular n-gon is

$$\frac{(n-2)\cdot 180^\circ}{n} = 180^\circ - \frac{360^\circ}{n}$$

To measure the exterior angles in a regular n-gon, notice that the interior angle and the corresponding adjacent exterior angle are supplementary. See Figure 40.6. Thus, the measure of each exterior angle is

$$180^\circ - \frac{(n-2)\cdot 180^\circ}{n} = \frac{360^\circ}{n}.$$

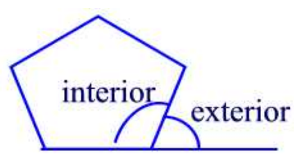


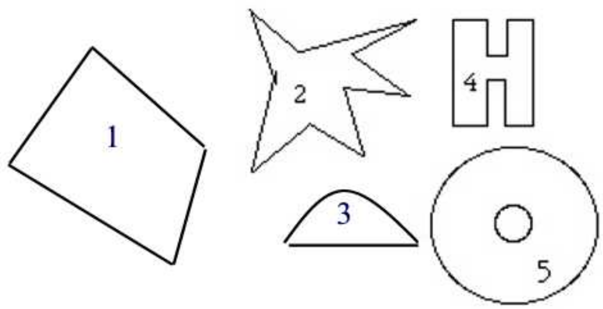
Figure 40.6

Example 40.1
(a) Find the measure of each interior angle of a regular decagon (i.e., n =10).
(b) Find the number of sides of a regular polygon, each of whose interior angles has a measure of 175°.

Solution.
(a) The measure of each angle is: $\frac{(10-2)\cdot 180^\circ}{10} = 144^\circ$.
(b) We are given that $\frac{(n-2)\cdot 180^\circ}{n} = 175^\circ$ or $180^\circ - \frac{360^\circ}{n} = 175^\circ$. This implies that $\frac{360^\circ}{n} = 180^\circ - 175^\circ = 5^\circ$. Thus, $n = \frac{360^\circ}{5^\circ} = 72$ ■

Practice Problems

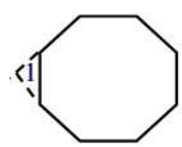
Problem 40.1
List the numerical values of the shapes that are convex .



Problem 40.2
Determine how many diagonals each of the following has:
(a) 20-gon (b) 100-gon (c) n-gon

Problem 40.3
In a regular polygon, the measure of each interior angle is 162°. How many sides does the polygon have?

Problem 40.4
Two sides of a regular octagon are extended as shown in the following figure. Find the measure of $\angle 1$.



Problem 40.5
Draw a quadrilateral that is not convex.

Problem 40.6
What is the sum of the interior angle measures of a 40-gon?

Problem 40.7
A Canadian nickel has the shape of a regular dodecagon (12 sides). How many degrees are in each interior angle?

Problem 40.8
Is a rectangle a regular polygon? Why or why not?

Problem 40.9
Find the measures of the interior, exterior, and central angles of a 12-gon.

Problem 40.10
Suppose that the measure of the interior angle of a regular polygon is $176°$. What is the measure of the central angle?

Problem 40.11
The measure of the exterior angle of a regular polygon is $10°$. How many sides does this polygon have?

Problem 40.12
The measure of the central angle of a regular polygon is $12°$. How many sides does this polygon have?

Problem 40.13
The sum of the measures of the interior angles of a regular polygon is $2880°$. How many sides does the polygon have?

Problem 40.14
How many lines of symmetry does each of the following have?

(a) a regular pentagon
(b) a regular octagon
(c) a regular hexagon.

Problem 40.15
How many rotational symmetry does a pentagon have?