

39 Symmetry of Plane Figures

In this section, we are interested in the symmetric properties of plane figures. By a **symmetry** of a plane figure we mean a motion of the plane that moves the figure so that it falls back on itself.

The two types of symmetry that we discuss are

Reflection Symmetry:

A plane figure is symmetric about a line if it is its own image when flipped across the line. We call the reflection line the **line of symmetry**. In other words, a figure has a line symmetry if it can be folded along the line so that one half of the figure matches the other half.

Reflection symmetry is also known as **mirror symmetry**, since the line of symmetry acts like a double-sided mirror. Points on each side are reflected to the opposite side.

Many plane figures have several line symmetries. Figure 39.1 shows some of the plane figures with their line symmetries shown dashed.

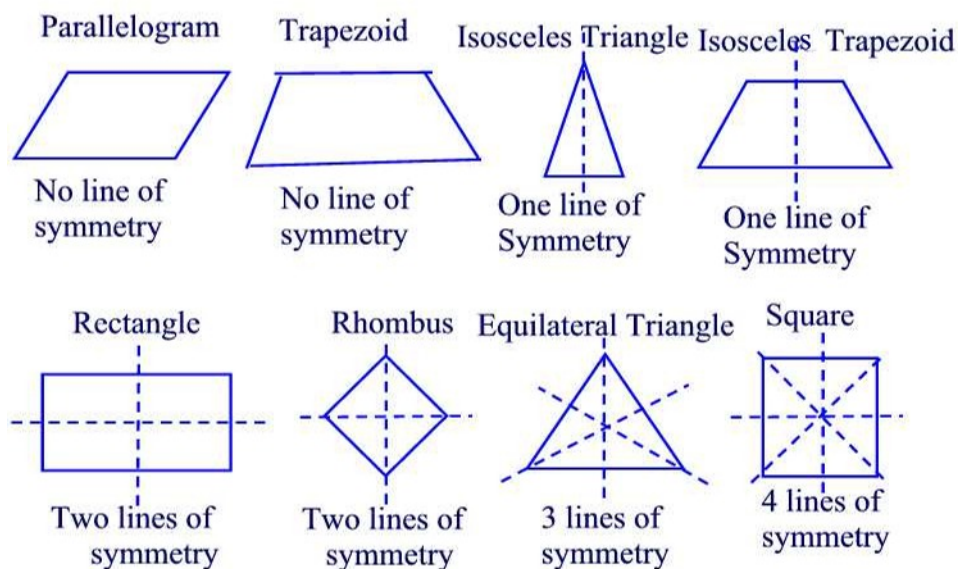


Figure 39.1

Remark 39.1

Using reflection symmetry we can establish properties for some plane figures. For example, since an isosceles triangle has one line of symmetry then the

base angles, i.e. angles opposed the congruent sides, are congruent. A similar property holds for isosceles trapezoids.

Rotational Symmetry:

A plane figure has **rotational symmetry** if and only if it can be rotated more than 0° and less than or equal to 360° about a fixed point called the **center of rotation** so that its image coincides with its original position. Figure 39.2 shows the four different rotations of a square.

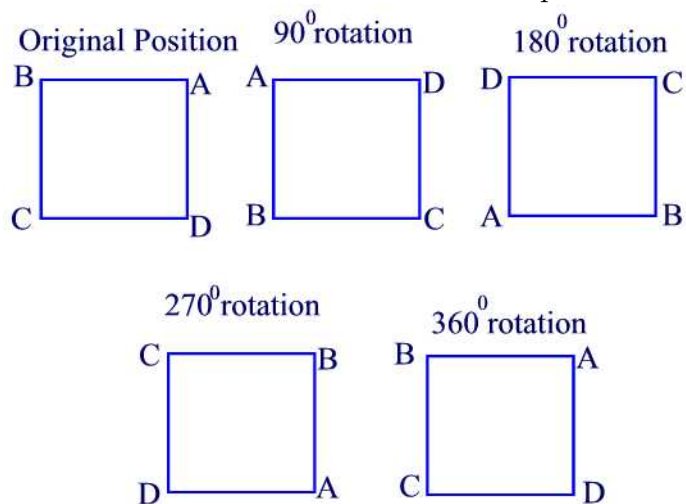


Figure 39.2

The following table list the number of turns, including a full turn, of some plane figures.

Figure	Rotation
Rectangle	2 rotations(180° and 360°)
Square	4 rotations(90° , 180° , 270° , 360°)
Rhombus	2 rotations(180° and 360°)
Parallelogram	2 rotations(180° and 360°)
Trapezoid	1 rotation(360°)
Equilateral Triangle	3 rotations(120° , 240° , and 360°)

Example 39.1

Show that the opposite sides of a parallelogram are congruent.

Solution.

Rotate the parallelogram 180° about its center as shown in Figure 39.3.

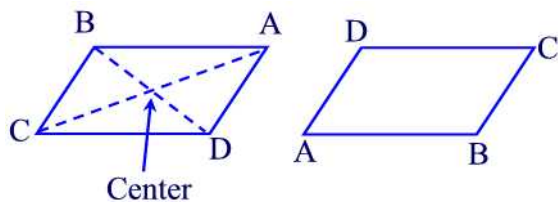


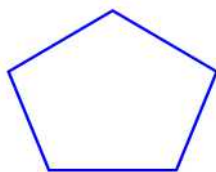
Figure 39.3

It follows that $\overline{AB} = \overline{DC}$ and $\overline{BC} = \overline{AD}$ ■

Practice Problems

Problem 39.1

Draw all lines of symmetry for the figure below.

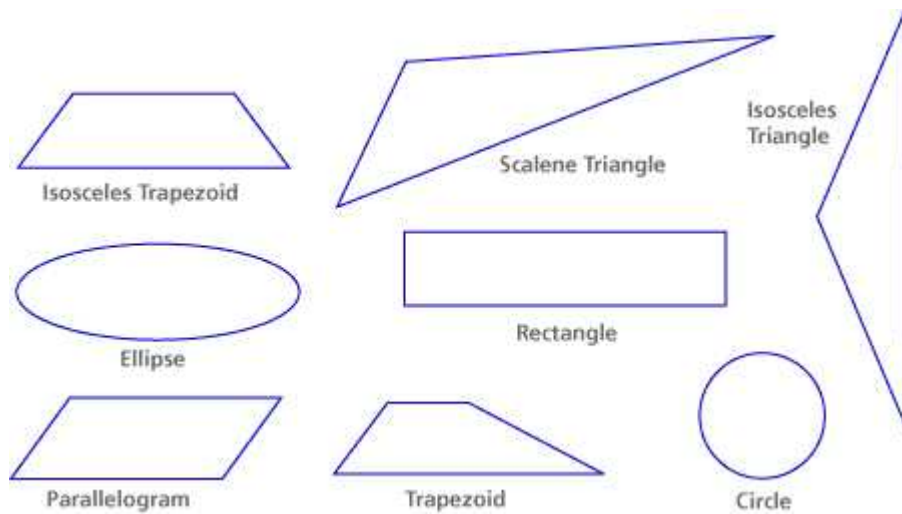


Problem 39.2

- (a) Find the vertical line(s) of symmetry of the letters A, U, V, T, Y.
- (b) Find the horizontal line(s) of symmetry of the letters D, E, C, B.
- (c) Find the vertical and horizontal line(s) of symmetry of the letters H, I, O, X.

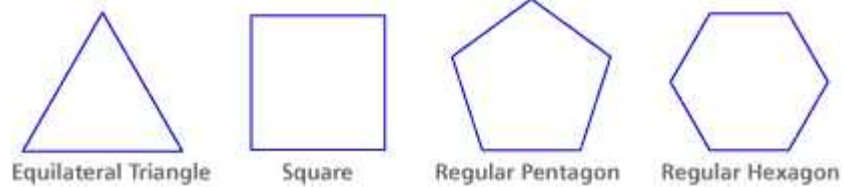
Problem 39.3

For each figure, find all the lines of symmetry you can.



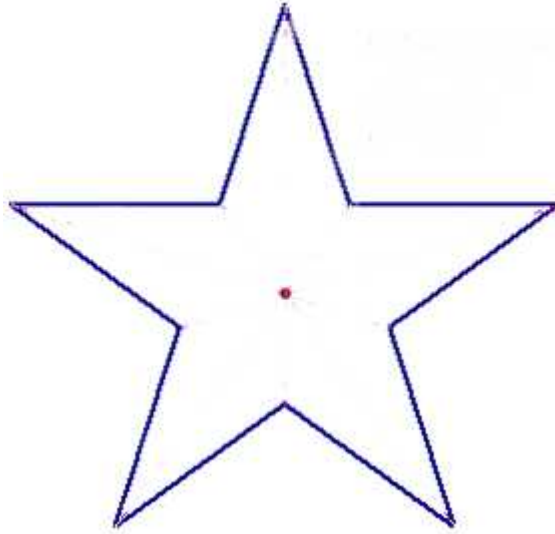
Problem 39.4

A regular polygon is a closed figure with all sides congruent. Find all the lines of symmetry for these regular polygons. Generalize a rule about the number of lines of symmetry for regular polygons.



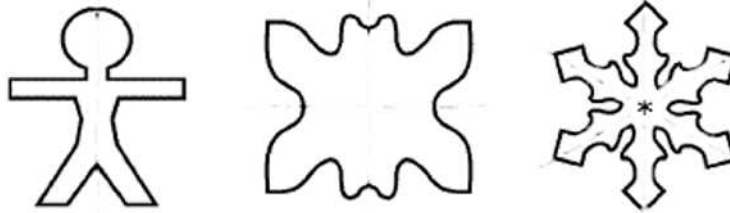
Problem 39.5

Find the number of rotations of the following geometric shape.



Problem 39.6

For each figure, find all the lines of symmetry you can.



Diagonals of Quadrilaterals

A **diagonal** of a quadrilateral is a line segment formed by connecting non-adjacent vertices (i.e., not on the same side). Each quadrilateral discussed in Section 37 has two diagonals.

Example 39.2

For each geoboard quadrilateral below draw in the two diagonals. Enter the data requested below each figure. The meaning of each entry is given here.

Type: Which of the seven types of quadrilaterals best describes the figure: square, rhombus, rectangle, parallelogram, kite, trapezoid or isosceles trapezoid.

Cong \cong : Are the diagonals congruent? Yes or No.

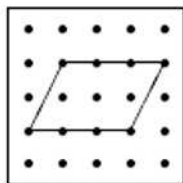
Perp \perp : Are the diagonals perpendicular? Yes or No.

Bisect: Do the diagonals bisect each other? Yes or No.

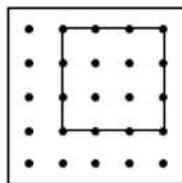
Bisect \sphericalangle : Do the diagonals bisect the corner angles? Yes or No.

Instructions: Complete this table by entering yes or no in each box.

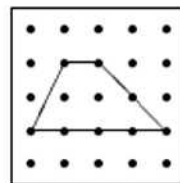
Quads	Diags. \cong	Diags \perp	Diags Bisect	Diags Bisect \sphericalangle
Square				
Rhombus				
Rectangle				
Parallelogram				
Kite				
Trapezoid				
Iso. Trap.				



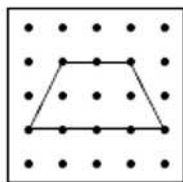
Type:
Cong \cong :
Perp \perp :
Bisect:
Bisect \sphericalangle :



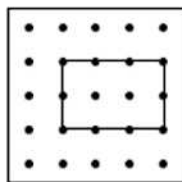
Type:
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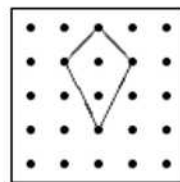
Type:
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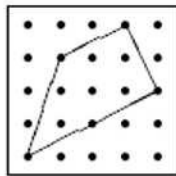
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Bisect \sphericalangle :



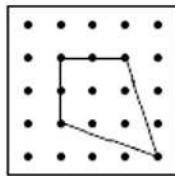
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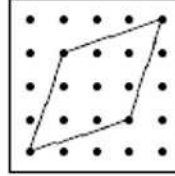
Type:
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Bisect \sphericalangle :



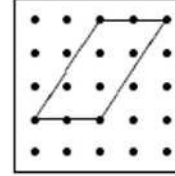
Type:
 Cong \cong :
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 Bisect:
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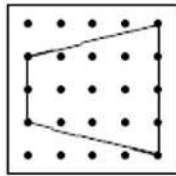
Type:
 Cong \cong :
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 Bisect:
 Bisect \angle :



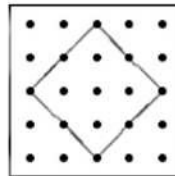
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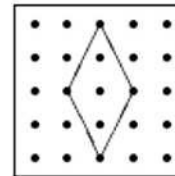
Type:
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 Bisect:
 Bisect \angle :



Type:
 Cong \cong :
 Perp \perp :
 Bisect:
 Bisect \angle :

Solution.

Quads	Diags. \cong	Diags \perp	Diags Bisect	Diags Bisect \angle
Square	yes	yes	yes	yes
Rhombus	yes	yes	yes	yes
Rectangle	yes	No	yes	No
Parallelogram	yes	No	yes	No
Kite	No	yes	No	No
Trapezoid	No	No	No	No
Iso. Trap.	yes	No	No	No

Circles

So far we have been studying plane figures with boundaries consisting of line segments. Circles are plane figures with curved boundary. By definition, a **circle** is the set of all points in the plane that are equidistant from a fixed

point called the **center** of the circle. The distance from the center to a point on the circle is called the **radius**. Any line segment crossing the center and whose endpoints are on the circle is called a **diameter** of the circle. It is clear that the length of a diameter is twice the length of the radius of the circle. See Figure 39.4.

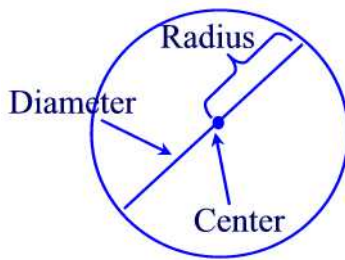


Figure 39.4

A useful device for drawing circles is the **compass** as shown in Figure 39.5.



Figure 39.5

The steps for drawing a circle with a compass are as follows:

1. Insert a sharp pencil into the holder on the compass.
2. Open the compass to the radius desired for your circle.
3. Place the compass point on a piece of paper where you would like the center point of the circle to be.
4. Place the point of the pencil on the paper.
5. Rotate the compass to mark a circle on the paper with the pencil. Draw without lifting the point of the compass off the paper.

Finally, we conclude this section by looking at the symmetry properties of circles. Any diameter is a line of symmetry of a circle so that a circle has infinitely many lines of symmetry. Also, a circle has infinitely many rotation symmetries, since every angle whose vertex is the center of the circle is an angle of rotation symmetry.

Practice Problems

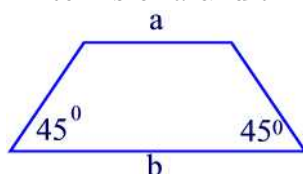
Problem 39.7

Let ABCD be a parallelogram.

- Prove that $\angle A$ and $\angle B$ are supplementary.
- Prove that angles $\angle A$ and $\angle C$ are congruent.

Problem 39.8

Using the figure below find the height of the trapezoid(i.e., the distance between the parallel sides) in terms of a and b .



Problem 39.9

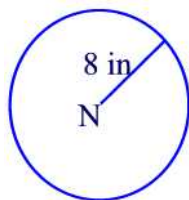
"The diagonals of a rectangle are congruent." Why does this statement imply that the diagonals of a square must also be congruent?

Problem 39.10

You learn the theorem that the diagonals of a parallelogram bisect each other. What other quadrilaterals must have this property?

Problem 39.11

Fill in the blank to describe the following circle with center N. Circle N is the set of _____ in a plane that are _____ from _____.



Problem 39.12

A **chord** is a line segment with endpoints on a circle. True or false? Every chord is also a diameter of the circle.

Problem 39.13

The diameter of a circle divides its interior into two congruent regions. How can someone use this property in dividing a circular pizza or pie in half?

Problem 39.14

If possible draw a triangle and a circle that intersect at exactly
(a) one point (b) two points (c) three points (d) four points

Problem 39.15

If possible draw two parallelograms that intersect at exactly
(a) one point (b) two points (c) three points (d) four points

Problem 39.16

A quadrilateral has two right angles. What can you deduce about the measures of the other two angles?