26 Integers: Multiplication, Division, and Order

Integer multiplication and division are extensions of whole number multiplication and division. In multiplying and dividing integers, the one new issue is whether the result is positive or negative. This section shows how to explain the sign of an integer product or quotient using number line, patterns, and signed counters.

Multiplication of Integers Using Patterns

Consider the following pattern of equalities which are derived from using the repeated addition .

$$\begin{aligned} 4 \times 3 &= 12 \\ 4 \times 2 &= 8 \\ 4 \times 1 &= 4 \\ 4 \times 0 &= 0 \end{aligned}$$

Note that the second factors in the successive products decrease by 1 each time and that the successive results decrease by 4. If this pattern continues we obtain the following sequence of equalities.

$$\begin{aligned} 4 \times 3 &= 12 \\ 4 \times 2 &= 8 \\ 4 \times 1 &= 4 \\ 4 \times 0 &= 0 \\ 4 \times (-1) &= -4 \\ 4 \times (-2) &= -8 \\ 4 \times (-3) &= -12 \end{aligned}$$

Since these results agree with what would be obtained by repeated addition, the pattern of the product is an appropriate guide. In general, it suggests that

$$m \cdot (-n) = -(m \cdot n)$$

where m and n are positive integers. This says that the product of a positive integer times a negative integer is always a negative integer.

Using this result we can write the following pattern of equalities

$$\begin{aligned} 3 \times (-3) &= -9 \\ 2 \times (-3) &= -6 \\ 1 \times (-3) &= -3 \\ 0 \times (-3) &= 0 \end{aligned}$$

So we notice that when the first factors decrease by 1 the successive results increase by 3. Continuing the pattern we find

$$\begin{aligned} 3 \times (-3) &= -9 \\ 2 \times (-3) &= -6 \\ 1 \times (-3) &= -3 \\ 0 \times (-3) &= 0 \\ (-1) \times (-3) &= 3 \\ (-2) \times (-3) &= 6 \end{aligned}$$

In general, this suggests that

$$(-m) \cdot (-n) = m \cdot n,$$

where m and n are positive integers. Hence the product of two negative integers is always positive.

What about the product of a negative integer times a positive integer such as $(-3) \times 2$? Using the results just derived above we see that

$$\begin{aligned} (-3) \times (-3) &= 9 \\ (-3) \times (-2) &= 6 \\ (-3) \times (-1) &= 3 \\ (-3) \times 0 &= 0 \end{aligned}$$

Note that the second factors in the successive products increase by 1 each time and the successive results decrease by 3. Hence, continuing the pattern we obtain

$$\begin{aligned} (-3) \times (-3) &= 9 \\ (-3) \times (-2) &= 6 \\ (-3) \times (-1) &= 3 \\ (-3) \times 0 &= 0 \\ (-3) \times 1 &= -3 \\ (-3) \times 2 &= -6 \end{aligned}$$

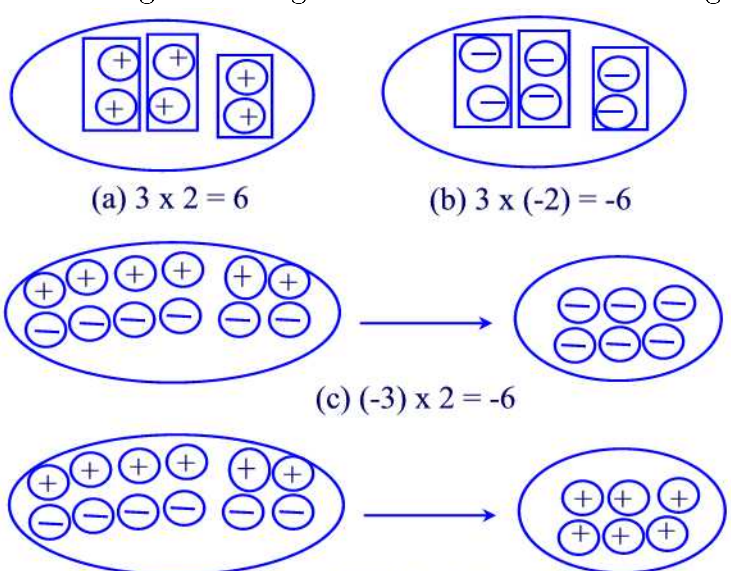
This suggests the general rule

$$(-n) \cdot m = -(n \cdot m),$$

where m and n are positive integers. That is, a negative number times a positive number is always negative.
Summarizing these results into a theorem we have

Theorem 26.1
Let m and n be two positive integers. Then the following are true:
(1) $m(-n) = -mn$
(2) $(-m)n = -mn$
(3) $(-m)(-n) = mn$.

Multiplication of Integers Using Signed Counters
The signed counters can be used to illustrate multiplication of integers, although an interpretation must be given to the sign. See Figure 26.1(a). The product 3×2 illustrates three groups each having 2 positive counters. A product such as $3 \times (-2)$ represents three groups each having 2 negative counters as shown in Figure 26.1(b). A product like $(-3) \times 2$ is interpreted as taking away 3 groups each having 2 positive counters as shown in Figure 26.1(c), and the product $(-3) \times (-2)$ is interpreted as taking away three groups each consisting of two negative counters as shown in Figure 26.1(d).



(a) 3 x 2 = 6
(b) 3 x (-2) = -6
(c) (-3) x 2 = -6
(d) (-3) x (-2) = 6

Figure 26.1

Multiplication of Integers Using Number Line
Since a product has a first factor and a second factor we can identify the first factor as the velocity of a moving object who starts from 0 and the second factor is time. A positive velocity means that the object is moving east and a negative velocity means that the object is moving west. Time in future is denoted by a positive integer and time in the past is denoted by a negative integer.

Example 26.1
Use a number-line model to answer the following questions.
(a) If you are now at 0 and travel east at a speed of 50km/hr, where will you be 3 hours from now?
(b) If you are now at 0 and travel east at a speed of 50km/hr, where were you 3 hours ago?
(c) If you are now at 0 and travel west at a speed of 50km/hr, where will you be 3 hours from now?
(d) If you are now at 0 and travel west at a speed of 50km/hr, where were you 3 hours ago?

Solution.
Figure 26.2 illustrates the various products using a number-line model.■

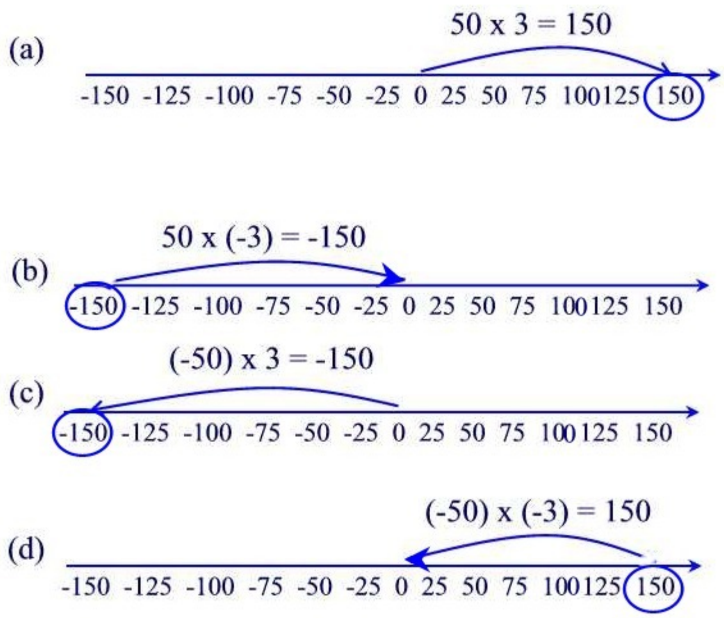


Figure 26.2

The set of integers has properties under multiplication analogous to those of the set of whole numbers. We summarize these properties in the following theorem.

Theorem 26.2
Let a, b, and c be any integers. Then the following are true.

Closure $a \cdot b$ is a unique integer.
Commutativity $a \cdot b = b \cdot a$
Associativity $a \cdot (b \cdot c) = (a \cdot b) \cdot c$
Identity Element $a \cdot 1 = 1 \cdot a = a$
Distributivity $a \cdot (b + c) = a \cdot b + a \cdot c$.
Zero Product $a \cdot 0 = 0 \cdot a = 0$.

Using the previous theorem one can derive some important results that are useful in computations.

Theorem 26.3
For any integer a we have $(-1) \cdot a = -a$.

Proof.
First, note that $0 = 0 \cdot a = [1 + (-1)] \cdot a = a + (-1) \cdot a$. But $a + (-a) = 0$ so that $a + (-a) = a + (-1)a$. By the additive cancellation property we conclude that $-a = (-1) \cdot a$ ■

We have seen that $(-a)b = -ab$ and $(-a)(-b) = ab$ where a and b are positive integer. The following theorem shows that those two equations are true for any integer not just positive integers.

Theorem 26.4
Let a and b be any integers. Then

$$(-a)b = -ab$$

and

$$(-a)(-b) = ab.$$

Proof.
To prove the first equation, we proceed as follows.

$$(-a)b = [(-1)a]b = (-1)(ab) = -ab.$$

To prove the second property, we have

$$(-a)(-b) = (-1)[a(-b)] = (-1)[(-a)b] = (-1)[(-1)(ab)] = (-1)(-1)(ab) = ab \blacksquare$$

Another important property is the so-called the **multiplicative cancellation property.**

Theorem 26.5
Let $a, b,$ and c be integers with $c \neq 0$. If $ac = bc$ then $a = b$.

Proof.
Suppose $c \neq 0$. Then either $c > 0$ or $c < 0$. If $\frac{c}{1} = c > 0$ then we know from Theorem 20.1 that c has a multiplicative inverse $\frac{1}{c}$, i.e. $cc^{-1} = 1$. In this case, we have

$$\begin{aligned} a &= a \cdot 1 \\ &= a(cc^{-1}) \\ &= (ac)c^{-1} \\ &= (bc)c^{-1} \\ &= b(cc^{-1}) \\ &= b \cdot 1 = b \end{aligned}$$

If $c < 0$ then $-c > 0$ and we can use the previous argument to obtain

$$\begin{aligned} a &= a \cdot 1 \\ &= a[(-c)(-c)^{-1}] \\ &= [a(-c)](-c)^{-1} \\ &= -(ac)(-c)^{-1} \\ &= -(bc)(-c)^{-1} \\ &= [b(-c)](-c)^{-1} \\ &= b[(-c)(-c)^{-1}] \\ &= b \cdot 1 = b \blacksquare \end{aligned}$$

The following result follows from the multiplicative cancellation property.

Theorem 26.6 *(Zero Divisor)*
Let a and b be integers such that $ab = 0$. Then either $a = 0$ or $b = 0$.

Proof.
Suppose that $b \neq 0$. Then $ab = 0 \cdot b$. By the previous theorem we must have $a = 0$ $\blacksquare$

Practice Problems

Problem 26.1
Use patterns to show that $(-1)(-1) = 1$.

Problem 26.2
Use signed counters to show that $(-4)(-2) = 8$.

Problem 26.3
Use number line to show that $(-4) \times 2 = -8$.

Problem 26.4
Change $3 \times (-2)$ into a repeated addition and then compute the answer.

Problem 26.5
(a) Compute $4 \times (-3)$ with repeated addition.
(b) Compute $4 \times (-3)$ using signed counters.
(c) Compute $4 \times (-3)$ using number line.

Problem 26.6
Show how $(-2) \times 4$ can be found by extending a pattern in a whole-number multiplication.

Problem 26.7
Mike lost 3 pounds each week for 4 weeks.
(a) What was the total change in his weight?
(b) Write an integer equation for this situation.

Problem 26.8
Compute the following without a calculator.
(a) $3 \times (-8)$
(b) $(-5) \times (-8) \times (-2) \times (-3)$

Problem 26.9
Extend the following pattern by writing the next three equations.

$$\begin{array}{rcl} 6 \times 3 & = & 18 \\ 6 \times 2 & = & 12 \\ 6 \times 1 & = & 6 \\ 6 \times 0 & = & 0 \end{array}$$

Problem 26.10
Find the following products.
(a) $6(-5)$ (b) $(-2)(-16)$ (c) $-(-3)(-5)$ (d) $-3(-7-6)$.

Problem 26.11
Represent the following products using signed counters and give the results.
(a) $3 \times (-2)$ (b) $(-3) \times (-4)$

Problem 26.12
In each of the following steps state the property used in the equations.

$$\begin{array}{rcl} a(b-c) & = & a[b+(-c)] \\ & = & ab + a(-c) \\ & = & ab + [-(ac)] \\ & = & ab - ac \end{array}$$

Problem 26.13
Extend the meaning of a whole number exponent

$$a^n = \underbrace{a \cdot a \cdot a \cdots a}_{n\ factors}$$

where a is any integer and n is a positive integer. Use this definition to find the following values.
(a) $(-2)^4$ (b) -2^4 (c) $(-3)^5$ (d) -3^5

Problem 26.14
Illustrate the following products on an integer number line.
(a) $2 \times (-5)$ (b) $3 \times (-4)$ (c) $5 \times (-2)$

Problem 26.15
Expand each of the following products.
(a) $-6(x+2)$ (b) $-5(x-11)$ (c) $(x-3)(x+2)$

Problem 26.16
Name the property of multiplication of integers that is used to justify each of the following equations.
(a) $(-3)(-4) = (-4)(-3)$
(b) $(-5)[(-2)(-7)] = [(-5)(-2)](-7)$
(c) $(-5)(-7)$ is a unique integer
(d) $(-8) \times 1 = -8$
(e) $4 \cdot [(-8) + 7] = 4 \cdot (-8) + 4 \cdot 7$

Problem 26.17
If $3x = 0$ what can you conclude about the value of x?

Problem 26.18
If a and b are negative integers and c is positive integer, determine whether the following are positive or negative.
(a) $(-a)(-c)$ (b) $(-a)(b)$ (c) $(c-b)(c-a)$ (d) $a(b-c)$

Problem 26.19
Is the following equation true? $a \cdot (b \cdot c) = (a \cdot b) \cdot (a \cdot c)$.

Division of Integers

Suppose that a town's population drops by 400 people in 5 years. What is the average population change per year? The answer is given by $-400 \div 5$, where the negative sign indicates a decrease in population. Such a problem requires division of integers which we dicuss next.

You recall the missing-factor approach for the division of whole numbers: we write $a \div b, b \neq 0$ to mean a unique whole number c such that $a = bc$. Division on the set of integers is defined analogously:

Let a and b be any integers with $b \neq 0$. Then $a \div b$ is the unique integer c, if it exists, such that $a = bc$.

Example 26.2

Find the following quotients, if possible.

(a) $12 \div (-4)$ (b) $(-12) \div 4$ (c) $(-12) \div (-4)$ (d) $7 \div (-2)$

Solution.

(a) Let $c = 12 \div (-4)$. Then $-4c = 12$ and consequently, $c = -3$.

(b) If $c = (-12) \div 4$ then $4c = -12$ and solving for c we find $c = -3$.

(c) Letting $c = (-12) \div (-4)$ we obtain $-4c = -12$ and consequently $c = 3$.

(d) Let $c = 7 \div (-2)$. Then $-2c = 7$ Since there is no integer when multiplied by -2 yields 7 then we say that $7 \div (-2)$ is undefined in the integers ■

The previous example suggests the following rules of division of integers.

(i) The quotient, if it exists, of two integers with the same sign is always positive.

(ii) The quotient, if it exists, of two integers with different signs is always negative.

Negative Exponents and Scientific Notation

Recall that for any whole number a and n, a positive integer we have

$$a^n = \underbrace{a \cdot a \cdot a \cdots a}_{n \ factors}.$$

We would like to extend this definition to include negative exponents. This is done by using "looking for a pattern" strategy. Consider the following sequence of equalities where a is a non zero integer.

$$\begin{array}{rcl} a^3 & = & a\cdot a\cdot a \\ a^2 & = & a\cdot a \\ a^1 & = & a \\ a^0 & = & 1 \end{array}$$

We see that each time the exponent is decreased by 1 the result is being divided by a. If this pattern continues we will get, $a^{-1} = \frac{1}{a}, a^{-2} = \frac{1}{a^2}$, etc. In general, we have the following definition.
Let $a \neq 0$ be any integer and n be a positive integer then

$$a^{-n} = \frac{1}{a^n}.$$

It can be shown that the theorems on whole-number exponents discussed in Section 10 can be extended to integer exponents. We summarize these properties in the following theorem.

Theorem 26.7
Let a, b be integers and m, n be positive integers. Then
(a) $a^m \cdot a^n = a^{m+n}$
(b) $\frac{a^m}{a^n} = a^{m-n}$
(c) $(a^m)^n = a^{mn}$
(d) $(a\cdot b)^m = a^m \cdot b^m$
(e) $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$, assuming $b \neq 0$
(f) $\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m$.

Example 26.3
Write each of the following in simplest form using positive exponents in the final answer.
(a) $16^2 \cdot 8^{-3}$ (b) $20^2 \div 2^4$ (c) $(10^{-1} + 5\cdot 10^{-2} + 3\cdot 10^{-3})\cdot 10^3$

Solution.
(a) $16^2 \cdot 8^{-3} = (2^4)^2 \cdot (2^3)^{-3} = 2^8 \cdot 2^{-9} = 2^{8-9} = 2^{-1} = \frac{1}{2}$.
(b) $20^2 \div 2^4 = (5\cdot 2^2)^2 \div 2^4 = (5^2 \cdot (2^2)^2) \div 2^4 = (25 \cdot 2^4) \div 2^4 = 5^2 = 25$.
(c)

$$\begin{array}{rcl} (10^{-1} + 5\cdot 10^{-2} + 3\cdot 10^{-3})\cdot 10^3 & = & 10^{-1}\cdot 10^3 + 5\cdot 10^{-2}\cdot 10^3 + 3\cdot 10^{-3}\cdot 10^3 \\ & = & 10^{-1+3} + 5\cdot 10^{-2+3} + 3\cdot 10^{-3+3} \\ & = & 10^2 + 5\cdot 10 + 3 \\ & = & 153 \blacksquare \end{array}$$

In application problems that involve very large or very small numbers, we use scientific notation to represent these numbers. A number is said to be in **scientific notation** when it is expressed in the form $a \times 10^n$ where $1 \leq a < 10$ is called the **mantissa** and n is an integer called the **characteristic**. For example, the diameter of Jupiter in standard notation is 143,800,000 meters. Using scientific notation this can be written in the form 1.438×10^8 meters. When converting from standard notation to scientific notation and vice versa recall the following rules of multiplying by powers of 10: When you multiply by 10^n move the decimal point n positions to the right. When you multiply by 10^{-n} move the decimal point n positions to the left.

Example 26.4
Convert as indicated.
(a) 38,500,000 to scientific notation
(b) 4.135×10^{11} to standard notation
(c) 7.2×10^{-14} to standard notation
(d) 0.0000961 to scientific notation.

Solution.
(a) $38,500,000 = 3.85 \times 10^7$
(b) $4.135 \times 10^{11} = 413,500,000,000$
(c) $7.2 \times 10^{-14} = 0.000000000000072$
(d) $0.0000961 = 9.61 \times 10^{-5}$ ■

Order of Operations on Integers
When addition, subtraction, multiplication, division, and exponentiation appear without parentheses, exponentiation is done first, then multiplication and division are done from left to right, and finally addition and subtraction from left to right. Arithmetic operations that appear inside parentheses must be done first.

Example 26.5
Evaluate each of the following.
(a) $(2-5) \cdot 4 + 3$
(b) $2 + 16 \div 4 \cdot 2 + 8$
(c) $2 - 3 \cdot 4 + 5 \cdot 2 - 1 + 5$.

Solution.
(a) $(2-5) \cdot 4 + 3 = (-3) \cdot 4 + 3 = -12 + 3 = -9$.

(b) $2 + 16 \div 4 \cdot 2 + 8 = 2 + 4 \cdot 2 + 8 = 2 + 8 + 8 = 18.$
(c) $2 - 3 \cdot 4 + 5 \cdot 2 - 1 + 5 = 2 - 12 + 10 - 1 + 5 = 4.$ ■

Practice Problems

Problem 26.20
Perform these divisions.
(a) $36 \div 9$ (b) $(-36) \div 9$ (c) $36 \div (-9)$ (d) $(-36) \div (-9)$ (e) $165 \div (-11)$
(f) $275 \div 11$

Problem 26.21
Write another multiplication equation and two division equations that are equivalent to the equation

$$(-11) \cdot (-25,753) = 283,283.$$

Problem 26.22
Write two multiplication equations and another division equation that are equivalent to the equation

$$(-1001) \div 11 = -91.$$

Problem 26.23
Use the definition of division to find each quotient, if possible. If a quotient is not defined, explain why.
(a) $(-40) \div (-8)$ (b) $(-143) \div 11$ (c) $0 \div (-5)$ (d) $(-5) \div 0$ (e) $0 \div 0$

Problem 26.24
Find all integers x (if possible) that make each of the following true.
(a) $x \div 3 = -12$ (b) $x \div (-3) = -2$ (c) $x \div (-x) = -1$ (d) $0 \div x = 0$
(e) $x \div 0 = 1$

Problem 26.25
Write two division equations that are equivalent to $3 \times (-2) = -6$.

Problem 26.26
Explain how to compute $-10 \div 2$ using signed counters.

Problem 26.27
Rewrite each of the following as an equivalent multiplication problem, and give the solution.
(a) $(-54) \div (-6)$ (b) $32 \div (-4)$

Problem 26.28
A store lost \$480,000 last year.
(a) What was the average net change per month?
(b) Write an integer equation for this situation.

Problem 26.29
Compute the following, using the correct rules for order of operations.
(a) $-2^2 - 3$ (b) $-5 + (-4)^2 \times (-2)$

Problem 26.30
A stock change as follows for 5 days:-2,4,6,3,-1. What is the average daily change in price?

Problem 26.31
Compute: $-2 \div (-2) + (-2) - (-2)$

Problem 26.32
For what integers a and b does $a \div b = b \div a$?

Problem 26.33
Find each quotient, if possible.
(a) $[144 \div (-12)] \div (-3)$ (b) $144 \div [-12 \div (-3)]$

Problem 26.34
Compute the following writing the final answer in terms of positive exponents.
(a) $4^{-2} \cdot 4^6$
(b) $\frac{6^3}{6^{-7}}$
(c) $(3^{-4})^{-2}$

Problem 26.35
Express each of the following in scientific notation.
(a) 0.0004 (b) 0.0000016 (c) 0.000000000000071

Problem 26.36
Hair on the human body can grow as fast as 0.0000000043 meter per second.
(a) At this rate, how much would a strand of hair grow in one month of 30 days? Express your answer in scientific notation.
(b) About how long would it take for a strand of hair to grow to be 1 meter in length?

Problem 26.37
Compute each of these to three significant figures using scientific notation.
(a) $(2.47 \times 10^{-5}) \cdot (8.15 \times 10^{-9})$
(b) $(2.47 \times 10^{-5}) \div (8.15 \times 10^{-9})$

Problem 26.38
Convert each of the following to standard notation.
(a) 6.84×10^{-5} (b) 3.12×10^{7}

Problem 26.39
Write each of the following in scientific notation.
(a) 413,682,000 (b) 0.000000231 (c) 100,000,000

Problem 26.40
Evaluate each of the following.
(a) $-5^2 + 3(-2)^2$
(b) $-2 + 3 \cdot 5 - 1$
(c) $10 - 3 \cdot 7 - 4(-2) \div 2 + 3$

Problem 26.41
Evaluate each of the following, if possible.
(a) $(-10 \div (-2))(-2)$
(b) $(-10 \cdot 5) \div 5$
(c) $-8 \div (-8 + 8)$
(d) $(-6 + 6) \div (-2 + 2)$
(e) $|-24| \div 4 \cdot (3 - 15)$

Comparing and Ordering Integers
In this section we extend the notion of "less than" to the set of all integers. We describe two equivalent ways for viewing the meaning of less than: a

number line approach and an addition (or algebraic) approach. In what follows, a and b denote any two integers.

Number-Line Approach

We say that a is **less than** b, and we write $a < b$, if the point representing a on the number-line is to the left of b. For example, Figure 26.3 shows that $-4 < 2$.

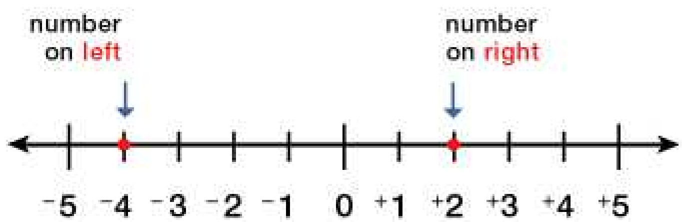


Figure 26.3

Example 26.6

Order the integers $-1, 0, 4, 3, -2, 2, -4, -3, 1$ from smallest to largest using the number line approach.

Solution.

The given numbers are ordered as shown in Figure 26.4■

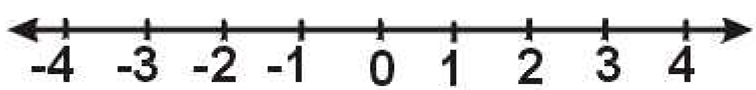


Figure 26.4

Addition Approach

Note that from Figure 26.3 we have $2 = (-4) + 6$ so we say that 2 is 6 more than -2. In general, if $a < b$ then we can find a unique integer c such that $b = a + c$.

Example 26.7

Use the addition approach to show that $-7 < -3$.

Solution.

Since $-3 = (-7) + 4$ then -3 is 4 more than -7, that is $-7 < -3$■

Notions similar to less than are included in the following table.

Inequality Symbol	Meaning
$<$	less than
$>$	greater than
$\leq$	less than or equal
$\geq$	greater than or equal

The following rules are valid for any of the inequality listed in the above table.

Rules for Inequalities
• **Trichotomy Law:** For any integers a and b exactly one of the following is true:

$$a < b, a > b, a = b.$$

• **Transitivity:** For any integers a, b, and c if $a < b$ and $b < c$ then $a < c$.
• **Addition Property:** For any integers a, b, and c if $a < b$ then $a+c < b+c$.
• **Multiplication Property:** For any integers a, b, and c if $a < b$ then $ac < bc$ if $c > 0$ and $ac > bc$ if $c < 0$.

The first three properties are extensions of similar statements in the whole number system. Note that the fourth property asserts that an inequality symbol must be reversed when multiplying by a negative integer. This is illustrated in Figure 26.5 when $c = -1$.

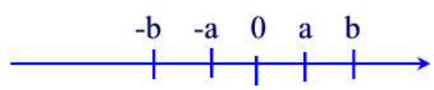


Figure 26.5

Practice Problems

Problem 26.42
Use the number-line approach to verify each of the following.
(a) $-4 < 1$ (b) $-4 < -2$ (c) $-1 > -5$

Problem 26.43
Order each of the following lists from smallest to largest.
(a) $\{-4, 4, -1, 1, 0\}$
(b) $\{23, -36, 45, -72, -108\}$

Problem 26.44
Replace the blank by the appropriate symbol.
(a) If $x > 2$ then $x + 4$____6
(b) If $x < -3$ then $x - 6$____-9

Problem 26.45
Determine whether each of the following statements is correct.
(a) $-3 < 5$ (b) $6 < 0$ (c) $3 \leq 3$ (d) $-6 > -5$ (e) $2 \times 4 - 6 \leq -3 \times 5 + 1$

Problem 26.46
What different looking inequality means the same as $a < b$?

Problem 26.47
Use symbols of inequalities to represent "at most" and "at least".

Problem 26.48
For each inequality, determine which of the numbers $-5, 0, 5$ satisfies the inequality.
(a) $x > -5$ (b) $5 < x$ (c) $-5 > x$

Problem 26.49
Write the appropriate inequality symbol in the blank so that the two inequalities are true.
(a) If $x \leq -3$ then $-2x$____6
(b) If $x + 3 > 9$ then x____6

Problem 26.50
How do you know when to reverse the direction of an inequality symbol?

Problem 26.51
Show that each of the following inequality is true by using the addition approach.
(a) $-4 < -2$ (b) $-5 < 3$ (c) $-17 > -23$

Problem 26.52
A student makes a connection between debts and negative numbers. So the number -2 represents a debt of \$2. Since a debt of \$10 is larger than a debt of \$5 then the student writes $-10 > -5$. How would you convince him/her that this inequality is false?

Problem 26.53
At an 8% sales tax rate, Susan paid more than \$1500 sales tax when she purchased her new Camaro. Describe this situation using an inequality with p denoting the price of the car.

Problem 26.54
Show that if a and b are positive integers with $a < b$ then $a^2 < b^2$. Does this result hold for any integers?

Problem 26.55
Elka is planning a rectangular garden that is twice as long as it is wide. If she can afford to buy at most 180 feet of fencing, then what are the possible values for the width?