Arkansas Tech University
MATH 1203: Trigonometry
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5 More Results of Functions

In this section, we continue the discussions of some function-related concepts that are of interest in this course.

Piecewise-Defined Functions

Piecewise-defined functions are functions defined by different formulas for different intervals of the independent variable.

Example 5.1 *(The Absolute Value Function)*
(a) Show that the function $f(x) = |x|$ is a piecewise defined function.
(b) Graph $f(x)$.

Solution.
(a) The absolute value function $|x|$ is a piecewise defined function since

$$|x| = \begin{cases} x & \text{for } x \geq 0, \\ -x & \text{for } x < 0. \end{cases}$$

(b) The graph is given in Figure 5.1.■

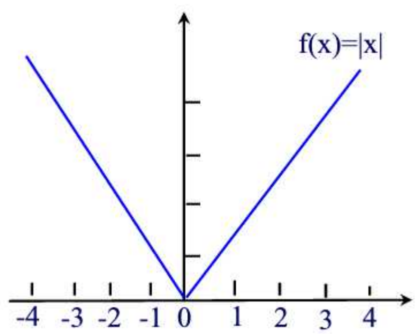

Figure 5.1

Example 5.2 *(The Ceiling Function)*
The Ceiling function $f(x) = \lceil x \rceil$ is the piecewise defined function given by

$$\lceil x \rceil = \text{smallest integer greater than or equal to x.}$$

Sketch the graph of $f(x)$ on the interval $[-3, 3]$.

Solution.
The graph is given in Figure 5.2. An open circle represents a point which is not included.■

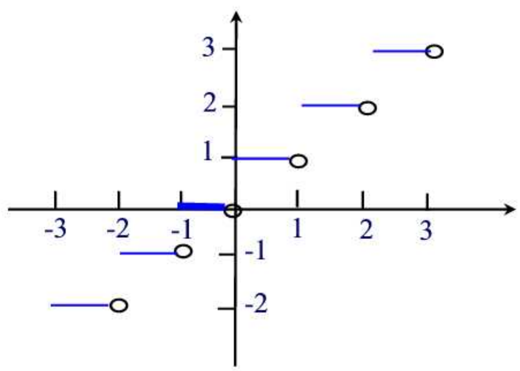

Figure 5.2

Example 5.3 *(The Floor Function)*
The Floor function $f(x) = \lfloor x \rfloor$ is the piecewise defined function given by

$$\lfloor x \rfloor = \text{greatest integer less than or equal to x.}$$

Sketch the graph of $f(x)$ on the interval $[-3, 3]$.

Solution.
The graph is given in Figure 5.3.■

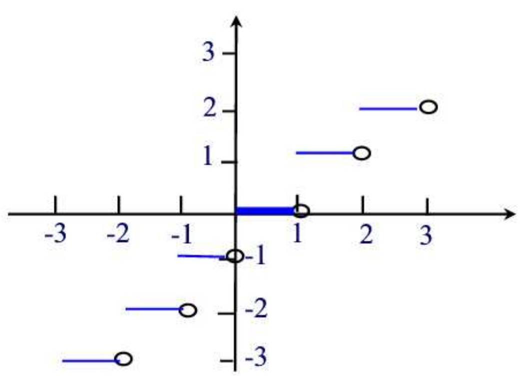

Figure 5.3

Example 5.4
Sketch the graph of the piecewise defined function given by

$$f(x) = \begin{cases} x+4 & \text{for } x \le -2, \\ 2 & \text{for } -2 < x < 2, \\ 4-x & \text{for } x \ge 2. \end{cases}$$

Solution.
The following table gives values of $f(x)$.

x	-3	-2	-1	0	1	2	3
f(x)	1	2	2	2	2	2	1

The graph of the function is given in Figure 5.4.■

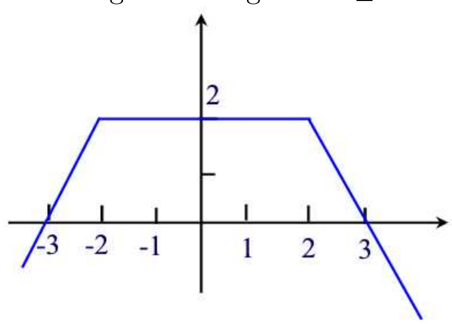

Figure 5.4

We next give a real-world situation where piecewise functions can be used.

Example 5.5
The charge for a taxi ride is \$1.50 for the first $\frac{1}{5}$ of a mile, and \$0.25 for each additional $\frac{1}{5}$ of a mile (rounded up to the nearest $\frac{1}{5}$ mile).

(a) Sketch a graph of the cost function C as a function of the distance traveled x, assuming that $0 \le x \le 1$.
(b) Find a formula for C in terms of x on the interval $[0, 1]$.
(c) What is the cost for a $\frac{4}{5} - mile$ ride?

Solution.
(a) The graph is given in Figure 5.5.

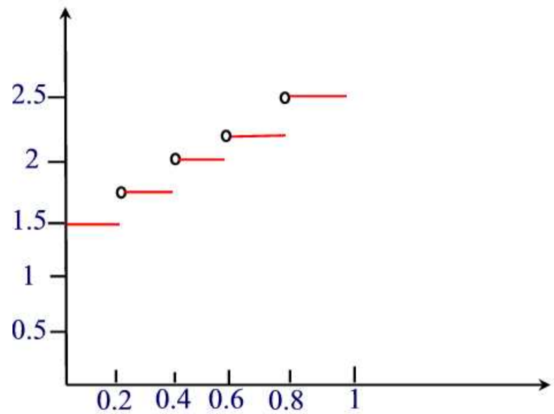

Figure 5.5

(b) A formula of $C(x)$ is

$$C(x) = \begin{cases} 0 & \text{if } x = 0 \\ 1.50 & \text{if } 0 < x \leq \frac{1}{5}, \\ 1.75 & \text{if } \frac{1}{5} < x \leq \frac{2}{5}, \\ 2.00 & \text{if } \frac{2}{5} < x \leq \frac{3}{5}, \\ 2.25 & \text{if } \frac{3}{5} < x \leq \frac{4}{5}, \\ 2.50 & \text{if } \frac{4}{5} < x \leq 1. \end{cases}$$

(c) The cost for a $\frac{4}{5}$ ride is $C(\frac{4}{5}) = \$2.25$.■

Increasing and Decreasing Functions
We say that a function is **increasing** if its graph climbs as x moves from left to right. That is, the function values increase as x increases. It is said to be **decreasing** if its graph falls as x moves from left to right. This means that the function values decrease as x increases.

Example 5.6
Determine the intervals where the function, given in Figure 5.6, is increasing and decreasing.

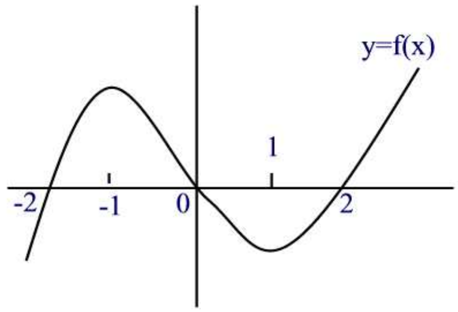

Figure 5.6

Solution.
The function is increasing on $(-\infty,-1)\cup(1,\infty)$ and decreasing on the interval $(-1,1)$.$\blacksquare$

One-To-One Functions
We have seen that when every vertical line crosses a curve at most once then the curve is the graph of a function f. We called this procedure the **vertical line test.** Now, if every horizontal line crosses the graph at most once then the function is called **one-to-one**.

Remark 5.1
The test used to identify one-to-one functions which we discussed above is referred to as the **horizontal line test**.

Example 5.7
Use a graphing calculator to decide whether or not the function is one-to-one.

(a) $f(x)=x^3+7$. (b) $g(x)=|x|$.

Solution.
(a) Using a graphing calculator, the graph of $f(x)$ is given in Figure 5.7. We see that every horizontal line crosses the graph once so the function is one-to-one.
(b) The graph of $g(x)=|x|$ (See Figure 5.1) shows that there are horizontal lines that cross the graph twice so that g is not one-to-one.$\blacksquare$

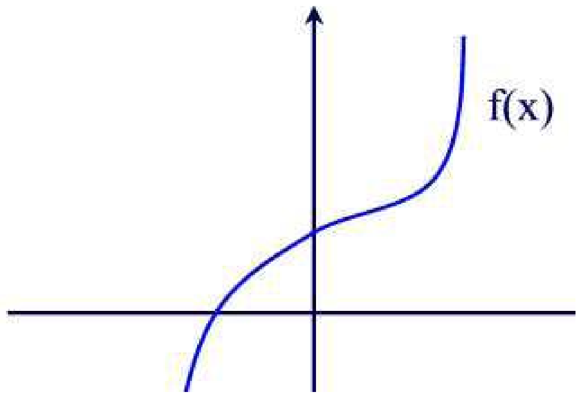

Figure 5.7