Arkansas Tech University
MATH 1203: Trigonometry
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2 Solving Algebraic Inequalities

This section illustrates the processes of solving linear, quadratic, and absolute value inequalities.

Solving Linear Inequalities
By a **linear inequality** we mean an inequality of the form

$$ax + b \,\square\, 0, \quad a \neq 0$$

where $\square$ can be any of the following:$<, >, \leq, \geq$.
To isolate the x, use the following two properties:

Property III: Adding or subtracting the same number to both sides of an inequality does not change the solution to the inequality.
Property IV: Multiplying or dividing both sides of an equality by a nonzero number does not change the solution to the inequality. However, when you multiply or divide by a negative number make sure you reverse the inequality sign.

Example 2.1
Solve the inequality: $x + 4 > 3x + 16$.

Solution.
Add $-x-16$ to both sides of the inequality to obtain $-12 > 2x$ or $2x < -12$. Now divide both sides by 2 to obtain $x < -6$. The solution set is usually represented by an interval. Thus, the interval of solution to the given inequality is $(-\infty, -6)$■

Solving Quadratic Inequalities
By a **quadratic inequality** we mean an inequality of the form

$$ax^2 + bx + c \,\square\, 0, \quad a \neq 0$$

where $\square$ can be any of the following:$<, >, \leq, \geq$.
The process of solving this type of inequalities consists of factoring the quadratic expressions so that we can locate the zeros and then construct

a chart of signs which provide the solution interval to the inequality. This process of solution is referred to as the **critical value method.**We illustrate this in the next example.

Example 2.2
Solve the inequality $6x^2 - 4 \leq 5x$.

Solution.
Subtract $5x$ from both sides to obtain $6x^2 - 5x - 4 \leq 0$. Factor $f(x) = 6x^2 - 5x - 4 = (3x - 4)(2x + 1)$. Thus, the zeros (also known as the **critical values**) of the left-hand side are $x = \frac{4}{3}$ and $x = -\frac{1}{2}$. These values separate the real line into three pieces. In each piece we select a **test value** to find the sign of f. Since $f(-1) = 7 > 0,\ f(0) = -4 < 0$, and $f(2) = 10 > 0$, we can construct the following chart of signs:

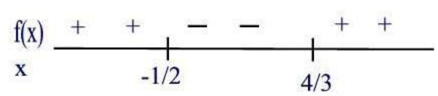

Figure 2.1

According to Figure 2.1, the interval of solution is given by $[-\frac{1}{2}, \frac{4}{3}]$.■

Solving Absolute Value Inequalities
Recall from Section 1, the definition of the **absolute value** of a number x

$$|x| = \begin{cases} x, & \text{if } x \geq 0, \\ -x, & \text{if } x < 0. \end{cases}$$

Geometrically, $|x|$ measures the distance from x to the origin. Thus, an inequality of the form $|x| > 5$ indicates that x is more than five units from 0. Any number on the number line to the right of 5 or to the left of -5 is more than five units from 0. So $|x| > 5$ is equivalent to $x < -5$ or $x > 5$. Thus, the interval of solution is given by the union of the intervals $(-\infty, -5)$ and $(5, \infty)$. Symbolically, we will write $(-\infty, -5) \cup (5, \infty)$.
Similarly, the inequality $|x - 9| < 2$ indicates that the distance from x to 9 is less than 2. On a number line, this happens when x is between 7 and 11. That is, the interval of solution is $(7, 11)$. As you can see, drawing the real line plays a major role in understanding how the solution interval(s) can be found.

Example 2.3
Solve $|5 - 3x| \leq 6$.

Solution.
Let $u = 5-3x$. Then $|u| \leq 6$. This means that the distance from u to 0 is less than or equal to 6. On a number line, this happens when $-6 \leq u \leq 6$. Thus, $-6 \leq 5-3x \leq 6$. Next, we have to isolate the x in this compound inequality. Subtract 5 from each part of the inequality to obtain $-11 \leq -3x \leq 1$. Now, divide through by -3 to obtain $-\frac{1}{3} \leq x \leq \frac{11}{3}$. Thus, the interval of solution is $[-\frac{1}{3}, \frac{11}{3}]$.■